

Near-threshold Z-pair production in the model of unstable particles with smeared mass

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Abstract

Near-threshold production of neutral boson pairs is considered within the framework of the model of unstable particles with smeared mass. The results of calculations are in good agreement with LEP II data and Monte-Carlo simulations. Suggested approach significantly simplifies calculations with respect to standard perturbative one.

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I. INTRODUCTION

Theoretical description of the near-threshold Z - and W -pair production in channels $e^+e^- \rightarrow ZZ, WW$ became actual since LEP II operation has started [1]-[8]. Now the interest to the boson-pair production remains topical as far as the boson intermediate states enter the high energy cascade processes at future hadron and linear colliders [9]-[12]. At threshold energy $\sqrt{s} \approx 2M_{Z,W}$ the finite-width effects (FWE) play a significant role in the boson-pair production. In the stable particle approximation (i.e. having bosons with zero widths) the on-shell boson-pair production has a non-physical threshold $\sqrt{s_{th}} = 2M$ which turns out to be eliminated (smeared) by taking into account of FWE. Usually it is fulfilled with help of the dressed boson propagator in processes like $e^+e^- \rightarrow ZZ \rightarrow 4f$ (Double Pole Approximation).

The calculation of the total cross-section $\sigma(e^+e^- \rightarrow 4f)$ including high-order perturbative corrections is very complicated problem and usually carried out by Monte-Carlo simulations [6, 8]. The so-called semi-analytical approach (SAA) significantly reduces the complexity of the calculations and gives the cross-sections in a quite compact analytical form [3]-[5]. This approach is based on the approximate factorization of the cross-section. In order to simplify the perturbative calculations and effectively incorporate the higher order corrections the effective theory of unstable particles (UP) was explored in [11, 12].

In this paper we consider Z -pair production within the framework of unstable particles model with a smeared mass [13, 14]. In the model UP is described as on-shell state having variable (smeared) mass in the vicinity of the threshold. The inclusive process $e^+e^- \rightarrow ZZ, WW \rightarrow all(4f)$ in Double Pole Approximation (DPA) is described as a boson-pair production process $e^+e^- \rightarrow ZZ, WW$ where bosons are on the fuzzed (smeared) mass-shell. This treatment is in close analogy with SAA, but our model contains new element - the smearing (spreading) of mass, which is directly connected with instability (see Section 2). In contrast to SAA or convolution method the model leads to exact factorization in wide class of processes [15, 16]. As was shown in [16], the model predictions at high energies have to coincide with standard perturbative results with very high accuracy. In this paper we have got the same coincidence while the calculations are much simpler.

The paper organized as follows. In Section 2 we briefly describe the UPs model and derive the double convolution formula for the Z -pair production cross-section. The strategy of calculations and results including the one-loop EW corrections are discussed in Section 3. We stressed out that our results precisely coincide with the corresponding Monte-Carlo simulation. In Section 4 we make the conclusion concerning the applicability of suggested method.

II. CROSS-SECTION OF Z -PAIR PRODUCTION IN THE MODEL OF UNSTABLE PARTICLES WITH SMEARED MASS

The model of UP with smeared mass is based on fundamental relation between lifetime of unstable state and spreading of energy level. As was noted by Matthews and Salam [17], an account of instability in wave function of UP leads to the spreading (smearing) of its mass. A wave function of UP in their rest system with an account of decay width $\Gamma = 1/\tau$

can be written in terms of its Fourier transform:

$$\psi(t) = \exp\{iMt - \Gamma|t|/2\} \rightarrow \frac{\Gamma}{2\pi} \int \frac{\exp\{-imt\}}{(m - M)^2 + \Gamma^2/4} dm. \quad (2.1)$$

Right-hand part of (2.1) may be interpreted as describing a distribution of mass values, with a spread, δm , related to the mean life $\delta\tau = 1/\Gamma$, by an uncertainty relation $\delta m \delta\tau \sim 1$. The generalization of the field wave function (2.1) can be represented in the form [14]:

$$\Psi(x) = \int \Psi(x, \mu) \omega(\mu) d\mu, \quad (2.2)$$

where $\Psi(x, \mu)$ is standard spectral component defining a particle with fixed mass squared $m^2 = \mu$ in the stable particle approximation (SPA) and $\omega(\mu)$ is some weight function formed by self-energy interactions of UP with vacuum fluctuations and decay products. This function describes the smeared (fuzzed) mass-shell of UP. So, the smearing of mass is caused, on the one hand by instability according to formal relation (2.1) and on the other hand by stochastic self-energy type interaction of unstable system with vacuum fluctuations.

The transition amplitude of the process with one UP is factorized as $A(\mu) = A^{st}(\mu)\omega(\mu)$ [14], where $A^{st}(\mu)$ is the standard amplitude in SPA. Then, the differential probability of transition $e^+e^- \rightarrow Z(\mu_1)Z(\mu_2)$ is

$$dP(\mu_1, \mu_2) = |A^{st}(\mu_1, \mu_2)|^2 \rho(\mu_1) \rho(\mu_2) d\mu_1 d\mu_2, \quad (2.3)$$

where $\rho(\mu) = |\omega(\mu)|^2$ is the probability density of the mass squared distribution. As a result we get the double convolution formula for the cross-section

$$\sigma(e^+e^- \rightarrow ZZ) = \int_{\mu_0}^s \int_{\mu_0}^{(\sqrt{s}-\sqrt{\mu_1})^2} \sigma^{st}(e^+e^- \rightarrow Z(\mu_1)Z(\mu_2)) \rho(\mu_1) \rho(\mu_2) d\mu_1 d\mu_2, \quad (2.4)$$

where $\mu_0 \sim m_f$ is the threshold, $\sigma^{st}(e^+e^- \rightarrow Z(\mu_1)Z(\mu_2))$ is the cross-section of $Z(\mu_1)$ and $Z(\mu_2)$ pair production in SPA. As usual, we introduce the factor 1/2 to take into account the "integral" identity of $Z(\mu_1)$ and $Z(\mu_2)$. This factor is necessary to satisfy the stable particle limit

$$\rho(\mu) \rightarrow \delta(\mu - M^2), \quad \sigma(e^+e^- \rightarrow ZZ) \rightarrow \sigma^{st}(e^+e^- \rightarrow Z(M)Z(M)). \quad (2.5)$$

and to describe correctly the exclusive processes such as $e^+e^- \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2$ and $e^+e^- \rightarrow f_1 \bar{f}_1 f_1 \bar{f}_1$.

The process under consideration is given by elementary Born cross-section in the standard form

$$\sigma^{st}(e^+e^- \rightarrow Z(\mu_1)Z(\mu_2)) = \frac{g^4(1 + 6c^2 + c^4)}{2^{10}\pi s \cos^4 \theta_W} \bar{\lambda}(\mu_1, \mu_2; s) f(\mu_1, \mu_2; s), \quad (2.6)$$

where $c = 1 - 4 \sin^2 \theta_W$ and g is the weak coupling constant. The functions $\bar{\lambda}(\mu_1, \mu_2; s)$ and $f(\mu_1, \mu_2; s)$ are defined by the following expressions

$$\bar{\lambda}(\mu_1, \mu_2; s) = \left[1 - 2 \frac{\mu_1 + \mu_2}{s} + \frac{(\mu_1 - \mu_2)^2}{s^2} \right]^{1/2} \quad (2.7)$$

and

$$f(\mu_1, \mu_2; s) = -1 + \frac{s^2 + (\mu_1 + \mu_2)^2}{s(s - \mu_1 - \mu_2)\bar{\lambda}(\mu_1, \mu_2; s)} \arctan \frac{s\bar{\lambda}(\mu_1, \mu_2; s)}{s - \mu_1 - \mu_2}. \quad (2.8)$$

The probability density $\rho(\mu)$ was defined in Ref. [14] by three different ways. We choose it in traditional Lorentzian form

$$\rho(\mu) = \frac{1}{\pi} \frac{\sqrt{\mu} \Gamma_Z(\mu)}{(\mu - M_Z^2)^2 + \mu \Gamma_Z^2(\mu)}, \quad (2.9)$$

where $\Gamma_Z(\mu)$ is μ -dependent total width of Z -boson. The expressions (2.4) – (2.9) define the cross-section of two-boson production and, consequently, the inclusive $4f$ -production in DPA. The results in close analogy with (2.4) and (2.9) arise when instability is described by decay factor $\Gamma|t|$ in operator function of final Z -boson states. This effect was described in [19] as "fuzzy mass shell" for Majorana neutrinos.

To get the cross-section of exclusive $4f$ -production we represent the total width $\Gamma_Z(\mu)$ as

$$\Gamma_Z(\mu) = \sum_k \Gamma(Z(\mu) \rightarrow f_k \bar{f}_k). \quad (2.10)$$

Then, we can represent the exclusive DPA cross-section in the form

$$\begin{aligned} \sigma_{DP}(e^+e^- \rightarrow Z(\mu_1)Z(\mu_2) \rightarrow f_k \bar{f}_k f_i \bar{f}_i) &= \int_{\mu_0}^s \int_{\mu_0}^{(\sqrt{s}-\sqrt{\mu_1})^2} \sigma^{st}(e^+e^- \rightarrow Z(\mu_1)Z(\mu_2)) \times \\ &\quad (2 - \delta_{ik}) \rho_Z^k(\mu_1) \rho_Z^i(\mu_2) d\mu_1 d\mu_2, \end{aligned} \quad (2.11)$$

where the partial distribution $\rho_Z^k(\mu)$ is defined as

$$\rho_Z^k(\mu) = \frac{1}{\pi} \frac{\sqrt{\mu} \Gamma(Z(\mu) \rightarrow f_k \bar{f}_k)}{(\mu - M_Z^2)^2 + \mu \Gamma_Z^2(\mu)}. \quad (2.12)$$

Note, the combinatorial factor $2 - \delta_{ik}$ together with above mentioned factor $1/2$ takes into account the identity/nonidentity of fermion pairs in the final state.

The expression (2.11) has a close analogy with one obtained in the semi-analytical approach [3], but the methodological status of these results are different. Suggested approach is based on the fundamental relation between instability and smearing of energy level (i.e. mass of UP). In the framework of the model, Eq. (2.11) is derived from the exclusive cross-section (2.4), which directly follows from the structure of the model [14]. The same form of exclusive cross-section arises in standard approach (SAA) as an approximation, which is usually called the "narrow-width approximation" [20]. Within the framework of UPs model [14] the factorization at tree level is exact due to specific model propagator of UP as it was shown in Refs. [15, 16]. The higher order electro-weak corrections, which play a significant role in the considering high-energy process [1] – [12], will be discussed in the next section.

III. ANALYSIS OF RESULTS

The Born cross-section of ZZ -pair production for Z -boson states with fixed mass (stable particle approximation) and smeared mass (model state of UP) are represented in Fig. 1 by dashed and solid lines, respectively. From this figure one can see that the FWE give a

significant contribution at the energy interval $2M_Z - 10 \text{ GeV} \lesssim \sqrt{s} \lesssim 2M_Z + 10 \text{ GeV}$, where the threshold smearing is noticeable. At higher energies these two lines asymptotically approach each other. Both the curves significantly exceed the LEP data because the higher order electro-weak corrections give large contribution and have to be taken into consideration [3].

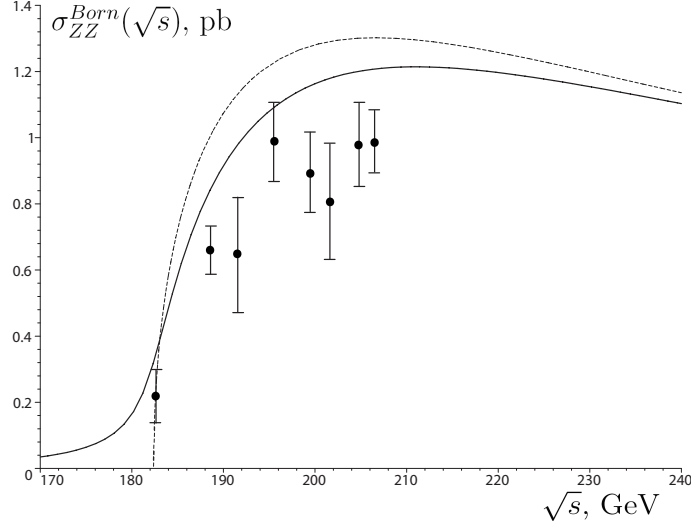


FIG. 1: Born ZZ cross-section in stable particle approximation (dashed line) and in the model of UP with smeared mass (solid line).

The strategy of calculations with taking into account of the higher order corrections is in the following. The self-energy corrections to the final states of unstable Z are incorporated in definition of the model wave function (2.2), i.e. in the weight function $\omega(\mu)$ or probability density $\rho(\mu)$. The principal part of vertex corrections is taken into account by using the weak running coupling $\alpha(M_Z) = 1/127.9$ [21]. The corrections to the initial and intermediate states of electron and positron caused by Initial State Radiation (ISR) and virtual radiation in the one-loop approximation were calculated before in Ref. [3], and we just follow the same procedure.

The model cross-section including above mentioned corrections is represented in Fig. 2 (the solid line) together with the result of Monte-Carlo simulation (the dashed line) and LEP data points [22]. Both results are consistent with the data within the error bars. We note that results of Monte-Carlo simulation and our model calculations coincide one with another with very high precision. From this result it follows that the contribution of non-factorizable corrections in the considered energy range is small. So, we can applied our approach to the process $e^+e^- \rightarrow W^+W^-$ in the same energy range. The lines start to differ slightly at energies larger than that of the available data, i.e. at $\sqrt{s} > 200 \text{ GeV}$. Note, that the difference between model and Monte-Carlo curves is an order of differences between results of various Monte-Carlo calculations.

The exclusive processes $e^+e^- \rightarrow q_i\bar{q}_i q_k\bar{q}_k$ at tree level are described by Eq. (2.11). For instance, it follows from Eq. (2.11) that

$$\frac{\sigma(e^+e^- \rightarrow q_i\bar{q}_i q_k\bar{q}_k)}{\sigma(e^+e^- \rightarrow q_i\bar{q}_i q_i\bar{q}_i)} \approx 2, \quad (3.1)$$

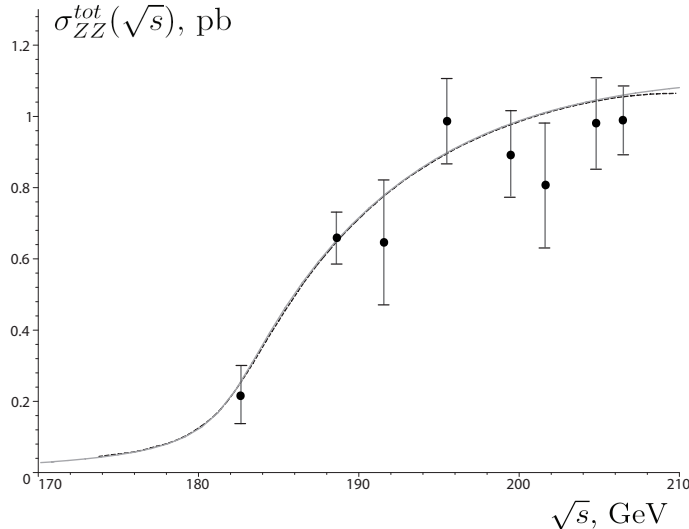


FIG. 2: Total ZZ cross-section obtained with Monte-Carlo simulations (dashed line) and in the model of UP with smeared mass (solid line).

and this result is in agreement with Monte-Carlo simulations [8]. It should be noted that the model assumes to account for non-factorizable corrections caused by interaction of initial states and final decay products of Z -bosons. In this case Z -boson in an intermediate state is described by model propagator [15, 16] and the convolution structure of cross-section is destructed. As was noted, such a correction is small in considered energy range.

IV. CONCLUSION

The calculation of $\sigma(e^+e^- \rightarrow 4f)$ together with the higher order electro-weak corrections is very complicated and longstanding problem intensified by nonfactorizable diagrams. Since the cross-sections are unavailable in an analytical form, the various approximations and Monte-Carlo simulations are usually applied. The semi-analytical approach allows to represent the exclusive cross-section in a compact analytical form and significantly simplifies inclusion of higher order corrections [3].

We have calculated the inclusive cross-section within the framework of the model of unstable particles with a smeared (random) mass. This approach has close analogy to the semi-analytical and convolution methods and, as a rule, gives the same results. However, our results are based on the simple model which explicitly describes the instability by smearing of mass and gives the cross-section and decay width in the convolution form. Moreover, as it was shown in Refs. [15, 16], the model provides an exact factorization in the cases when UP is in the intermediate state. So, this model can be treated as a methodological base for the SAA and CM. As was noted above, these approaches are valid for the discussed case giving quite accurate results.

The results of the model are in accordance with the experimental data and coincide with Monte-Carlo results with very high accuracy. In contrast to standard approach based on PT, the discussed method gives simple tools for calculations. It assumes an account of high-order corrections which in most cases do not destroy the convolution structure of cross-section and decay width.

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